MATH UP!

20 steps to conquer dyscalculia

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VU University Press

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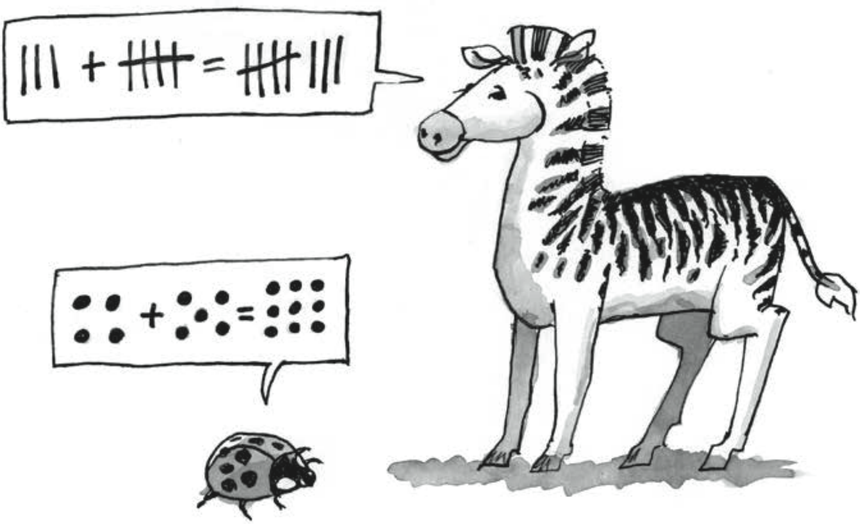

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INTRODUCTION

For a student with dyscalculia every step forward in arithmetic is a Herculean task. A procedure that is obvious or perhaps just a bit tricky for most pupils can be a serious obstacle for a math-disabled child. In this book I discuss the difficulties hidden within elementary arithmetic. No fractions, just counting up to 100. No long divisions, but simply memorizing the additions up to 10.

Dyscalculia is an impairment that occurs in different degrees of severity. In the DSM-5 dyscalculia is defined as a specific learning disorder, with an impairment of the learning of mathematics. Number sense, memorization of arithmetic facts, fluent calculation, and accurate math reasoning are named as problem areas.[1] The manual acknowledges three degrees of severity. This is a statistical arrangement. In reality, there is more variation.

The difficulties children meet on the road to math proficiency can be viewed as obstacles. Usually these obstacles are part of the numerical system itself, which is more complicated than one may think. Some obstacles are not directly related to arithmetic, but do influence a child's understanding of it. A weak visual-spatial imagination makes the understanding of arithmetic more difficult. And problems with word recognition and the discrimination of speech sounds (as is the case with dyslexia) complicate the process of learning multiplication tables.

This book, which was first published in Dutch in 2012, by Boom Publishers in Amsterdam, is the result of study and experience. The stories and examples are inspired by the work I do in collaboration with my husband Rob Milikowski, at our own practice De Rekencentrale and at schools in Amsterdam. These are real life stories, though the names of the children have been changed.

How to name the disability in question? In the book's title, I call it Dyscalculia. In the text I also may refer to it as a mathematical disability or an impairment. Children with dyscalculia I sometimes call dyscalculic but more often I call them math-disabled. In the scientific research literature all these terms may be encountered. And for my purpose in this book, they are all valid.

The obstacles are mostly presented in the order in which the student encounters them. Each obstacle has its own chapter, each with a similar structure. I present the difficulty or challenge, offer an example from the classroom, and then explain what helps, and what does not.

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Chapter 1
RECOGNITION

For children with dyscalculia, the regular laws of learning do not seem to apply. These laws suggest that practice makes perfect. But for a child with dyscalculia practice won't result in much progress, let alone perfection. This is strange, and people often don't believe it. They think the child in question simply hasn't had enough practice. That is what I used to think myself. But working with math-disabled children has taught me otherwise.

Amber was one of the six pupils I worked with at a primary school in Amsterdam: a diligent pupil, very keen to improve her command of math. At twelve years old, she was a self-assured and perfectly dressed lady. But when I gave her a speeded addition test, she was making elementary mistakes. For 4 + 3 she put down 8, and for 7 + 2 she wrote 10. I was astonished.

'Show me how you do it,' I said. '4 + 3: how do you figure that out?'

She put down her pencil and showed me her hand. 'Normally,' she said, 'I do it like this.' Starting from four, she counted the addition out on her fingers: '5, 6, 7.'

'Yes,' I said, 'that's right. But you wrote down 8.'

Amber looked at her work and pulled the sheet towards her. 'Can I correct it?'

Amber corrected it, but the correction was only on paper. In her head, the lack of precision remained. Amber's wish was to be able to work as quickly as the other children; to write down the answer directly and not have to bother with the stupid counting. If her classmates could do it, why couldn't she? It worked fine with geography and spelling, didn't it? But for her it didn't work with math. If she tried to come up with the answer without counting, she was often one place off. A sum such as 4 + 2 might result in 6, but it might just as well result in 7 or 5. Evidently, Amber could not rely on what her memory told her.

'But it is so obvious', Amber was often told in response to her mistakes. 'If only it was', she would think. 'If it really was obvious I wouldn't do it wrong.'

'But isn't it obvious?' That is what I also thought, the first time I worked with Amber. I was doing a test with her, meant for much younger pupils. One of the sums was 40 - 37. Amber stared at it and said: '17?'

I could understand how she got there. She first compared the tens and then the ones. The difference between 4 and 3 is 1. The difference between 7 and 0 is 7. A 1 and a 7, doesn't that make 17? Amber offered her solution with a question mark in her voice, because she *never* felt sure of herself when answering a math question.

'But Amber,' I said, 'don't you think 17 is a little too large for the difference between 40 and 37? How could 17 be made to fit into that small space?'

Amber looked at me without comprehension. 'What do you mean?'

'Well,' I said, 'if you count back from 40, when do you arrive at 37?' Amber counted back and established that she got there in three steps.

'Yes, that's right. But you said 17. That's a much bigger number.'

Amber grabbed her pencil. 'Can I write down the correct answer now?'

Looking back, I now see Amber as a child with dyscalculia. The errors made in simple sums, the imprecision whenever finger counting is abandoned: those are clear signals. The numbers we worked with are familiar territory for most twelve-year-olds. After six years of education, you shouldn't be making a mistake like 4 + 3 = 8 anymore. And if a child is making it nonetheless, he or she shouldn't be dismissed as stupid, lazy or careless. In such cases, something else is going on.

The insight that there is something like a learning disorder relating to arithmetic is a milestone in itself. By the recognition of dyscalculia a first obstacle on the path travelled by many children with problems in this area has been cleared. Two misunderstandings have long prevented the acceptance of dyscalculia.

The first misunderstanding occurs when dyscalculia is seen as a sure prediction of failure. Educational psychologist Annemie Desoete once published an article with the title: "Dyscalculia: there is at least one in every classroom."[2] This is not an outlandish estimate: the prevalence of dyscalculia is estimated at a minimum of 3 per cent, which works out at an average of one child in thirty-three. But many people were shocked by Desoete's suggestion. To them, 'dyscalculia' sounded as if a sentence was passed. As if the child in question was being written off. But the objective of assessing a child is not to label it. The aim is to get the math working again. That is only possible if you know what the obstacles are, and what help is needed to overcome them.

Another factor that has prevented the recognition and detection of dyscalculia is an educational model in which *understanding* is seen as the only accomplishment that counts. In this model, the process of developing automatisms is considered less important. One second slower or faster to solve a simple addition is not seen as significant. However, this underestimates the difficulties that children with dyscalculia struggle with. To be forced, time and again, to count out and calculate the most elementary number facts is not an enriching experience. It is a handicap.

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Chapter 2

REAL COUNTING

There is a large difference between reciting number words and counting.

A child who can recite the number words in the correct order isn't necessarily able to count. It is possible to learn the names of the numbers like words in a song, without properly understanding their meaning.

A child with a good memory for language will quickly memorize this particular song. She recites 'one, two, three, four, five, six, seven' and thinks that she is counting.

But counting is a more complex procedure. Truly being able to count means using the counting sequence to establish the number of objects in a set.

Look at these pencils on the table, you tell a child. How many do you think there are?

This is not a trivial question. A child of three years old may have a difficult time answering it. Three, six, four, eight: it all sounds pretty good. These are all possible answers to the question ‘how many?’. But what is the right choice between all those options?

A young child may think it is sufficient to recite a series of number words while pointing in the direction of the pencils. The mouth produces the string of words ‘onetwothreefourfivesixseven’ while the hand moves along the objects. But this is not how it should be done.

Counting is a *scientific* activity, the application of an ingenious system. For each consecutive object, you name the consecutive number. That is a difficult procedure for a toddler.

But when a child grasps the system, and when it can properly execute the procedure – pointing and saying 1, pointing and saying 2 – a new world is opened. From now on, there is no real limit to the reach of this tool. When you know enough counting words in the proper order, you can count *everything*. This is a powerful asset.[3]

Now let us see what Hafida, a young pupil of mine, can accomplish by counting.

Hafida is happy to play number games with the toy animals I show her: the cats Jake and Marge, and the duck called Cooper. All three are going to get some beads. Hafida is free to decide how many each animal will get, as long as the beads are fairly distributed. That is to say, each must have an equal number. Hafida promptly starts handing out beads to the animals. First each animal receives five. She is counting and verifying. That is fine. But then she loosely hands out some additional beads to each, and at a certain point she decides that it is enough. I see what she means, for in front of each animal there's a pretty pile of beads. Done, says Hafida.

Now I ask her to check how many beads each animal has actually received. Hafida first arranges the beads in a more orderly fashion. There, now she can count them. It turns out the two cats have nine beads each. It also becomes clear that the duck Cooper has been given eleven.

Cooper has too many. What can we do about that? First Hafida removes three beads from Cooper, but then quickly decides that this isn't right. She puts back the three beads. Then she removes one and leans back. This is the best she can do, for now. I propose that she count again how many beads Cooper has. Ten, Hafida establishes. Now she removes another bead and the job is done.

Hafida can count. She uses the proper method: pointing and naming. Point-and-name, point-and-name. Proceed like this until you have named all objects and you know *how many* there are, you know the number. Hafida has understood this well.[4]

If things get too messy to count precisely she arranges the beads in a row. Neatly. And she can apply the art of counting to accomplish a difficult task: to make things equal.

The next day Hafida approaches me in the schoolyard. When are we doing math again? I tell her that will be next week, and I ask if she remembers the names of the animals. With some help she remembers the three names. Next, I ask if she still remembers how many beads each one had. 'Nine', she says, 'and the duck had eleven.'

A checklist for counting proficiency

To ascertain that a child knows how to count in the range from 1 to 10, the following assignments can be used. This way you can find out what is still difficult for the child.

1. The number sequence.

- Counting forwards to 10
- Counting backwards from 10
- Counting on from any number, for instance: can you start counting at 3?
- Counting back from any number, for instance: can you count back from 6?
- Naming consecutives. What comes after 3? And after 7?
- Naming predecessors. What comes before 6? And before 9?

2. Counting objects

Put a row of blocks or beads in front of the child. You can also use other objects, for example teaspoons. How many are there? Start with a small number. And pay attention to the coordination of naming and indicating.

3. Counting out a certain number

Place some blocks on the table. Ask the child: can you give me 3 blocks? And now can you give me 5? In this task, coordination plays an important role again. But it also puts demands on working memory. Sometimes a child will lose track. 'What number did I just say?'

Pay close attention to the degree of effort it costs a child to execute these tasks. Parts that are still taxing should be regularly rehearsed. Otherwise, errors are bound to occur in a less focused situation.

4. Practicing

If there are hesitations and deficiencies, these parts should be taught and exercised again. Counting skills must become automatic. With sufficient training and support, every child can become a proficient counter. It just takes longer for some.

Finally, parents may practice these elements for a couple of minutes every day. In a light hearted way: with the stuffed animals, with the cutlery while laying the table, with the socks that come out of the laundry. And remember: don't make it too difficult. No adding or subtracting yet. The task is to count fluently in the range from one to ten.

3

Chapter 3

NUMBERS ON THEIR OWN FEET

Now a child knows the counting sequence, and he or she can also count out a number of objects. The next step is to know a number's place in the sequence *without* counting. This is more difficult than you may think, as the story of Yassin shows.

Yassin has to solve a problem: 3 + 2. He starts working on his left hand. One by one he raises his fingers: one, two, three. There, that makes three. Now he starts to work on his right hand. Plus two? Two fingers appear, first one, then another: one, two. So that's it. Now all he has to do is count the raised fingers on both hands. He does that by using his chin.

Yassin is eight years old and small for his age. He attends a school for special education. A lot of things are difficult for him, but that doesn't depress him. He doesn't mind solving math problems, as long as he understands what he is supposed to do. And he likes doing additions that fit on his hands.

But 6 + 2 is tricky, because a count of 6 does not fit on one hand. One, two, three, four, five – then what? The hand is full.

‘How does this work?’ he asks.

Perhaps, I think, Yassin can learn to do addition problems by *counting on*. He should be able to start at six and then count on for two more steps: seven, eight. So I take him into the hallway to do some walking math.

Again we start with 3 + 2. I choose a spot in the hallway and say: ‘Right, here is the three’.

But Yassin doesn’t see it this way. He lets go of my hand and walks back. Then he takes three steps forward: one, two, three. All right. Now he has actually reached the three. With this three, he can calculate. But not with mine. Because when I tell Yassin ‘three’, it’s only a word to him. To know what it means, how many it is, he has to count.

So now we take position on Yassin’s number three. ‘Why don’t you add two,’ I suggest. I expect him to count on: four, five.

But Yassin says: ‘one, two’.

Now we’re in trouble. Because where is the three we started from? It is nowhere to be found. And the two has disappeared as well. Why did we go to all that trouble? We have left precisely *nothing* to add up.

I make another feeble attempt to seduce Yassin to count two steps forward, starting from three. ‘Come on Yassin, we’re back on the three, look, here it was. And now we take two more steps: four, five!’

But Yassin doesn’t believe me anymore. If you take *two* steps, then you don’t say *four* and *five*! You say *one, two*, like he just showed me.

Thanks teacher, Yassin thinks, it was fun in the hallway, but now let me get back to my counting blocks, so I can do some proper math.

Five ways to solve an addition problem

How do young children solve a simple addition? American psychologist Robert Siegler found that children use five different strategies.[5] He asked kindergartners (5 to 6 years), first graders (6 to 7 years) and second graders (7 to 8 years) to solve the sum 5 + 3, and observed the way they went about it. These are the five strategies Siegler distinguished:

- **Counting all:** three fingers on one hand, five fingers on the other. Then count them together: 1, 2, 3, 4, 5, 6, 7, 8. This is Yassin's way.
- **Counting on:** Starting at 5, then counting on for three more steps: 6, 7, 8.
- **Using helpful knowledge:** 'I know that 4 + 4 = 8. I know that 3 is one less than 4, and 5 is one more. So 3 + 5 is equal to 4 + 4, and the answer is 8.'
- **Memory retrieval:** 8. 'I just know.'
- **Guessing:** 'I guessed.'

The three age groups showed a significant difference in the strategies they chose: the kindergartners solved the problem either by *counting all* or by *counting on*. Half of the second graders *just knew* the answer: they retrieved it from memory without counting. Most of the others solved the problem by counting on. In that grade the strategy of *counting all* was no longer used.

So, Yassin is still on the lowest rung of the ladder. The strategy he used to solve 3 + 2 = is the *counting all* strategy. More effective is *counting on*, the skill I tried to teach him in vain. Yassin was not yet able to make that leap. It would have been more effective if I'd played the game with him that I describe under 'EXTRA' at the end of this chapter.

Yassin is not an experienced counter yet. He must first learn to count easily and automatically. Formal additions can be tackled later. ‘Can you count up from three, Yassin? And from four?’ Then there’s counting backwards. Many children enjoy singing it out: Ten! Nine! Eight! And so on to the triumphant ZERO! It is a good thing, of course, when the reverse sequence has been mastered that way. But students also must be able to start at any given number in the reverse sequence also. ‘Can you count back from six? And from nine?’ All of this belongs to the foundation of arithmetic.

In a case like that of Yassin, dyscalculia is not the issue. His development is slow overall. But children with dyscalculia do face difficulties with mathematics that are similar to Yassin’s. To grasp the concepts of three, and five, and eight is difficult for them. It requires a lot of practice. In such cases it works best to practice a few minutes every day. Keep it simple and draw the child’s attention to the progress that has been made.

Practice

To help the child develop a firm grasp of the counting sequence from a given number, ask them to perform the following tasks:

- Counting forwards from two, from five, from seven
- Counting backwards from four, from three, from nine.
- Naming successors: Which number comes after three? And after seven?
- Naming predecessors: Which number comes before six? And before nine?

Don’t make it more difficult than that. Just keep practicing these simple skills a few minutes each day. Don’t be afraid that it will become boring. It’s pleasant for the child to learn to master this and to feel he is making progress.

Children don’t normally complain they are bored when they are mastering a skill. If they do say they are bored this often indicates the work is too difficult, that it goes over the child’s head. The resulting lack of meaning feels like boredom. The remedy is to retrace your steps and pick up the subject at a point where it becomes less 'boring'. That is, at the point corresponding to the child's level of understanding.

EXTRA: A game to practice.

Robert Siegler and his colleagues developed a simple board game with ten fields, numbered 1 through 10.[6] It is a good way for young children to master the counting sequence and to learn the correspondence between the words (e.g. four) and the digits (4).

How to play the Great Race? It's a teaching game for two, one adult and one child. There's no competition, at least not of the threatening kind. What is required is a die that only has the digits 1 and 2 on it, plus a playing board. This features a runway with the numbers 1 to 10. You also need two pawns. After throwing their die, players are allowed to move their pawn forward one or two steps, depending on the throw. The child must count out aloud the numbers he or she passes. Say: 'we're starting from position five. You have thrown a two? OK, your pawn can take two more steps. Let's read out the numbers. Six, seven, that is right. So you are on number seven now.'

The player to arrive at 10 first is the winner. It takes only a few minutes for a player to get there. If you are lucky and keep throwing two's, you may get there in five or six throws. It seems so simple. But let's not underestimate the knowledge that is being acquired. A child who plays this game learns the difference between numbers through experience:-

- The larger a number, the more steps it takes to reach it
- The larger a number, the longer the row of number words leading up to it
- The larger a number, the more time it takes to get there
- The larger a number, the greater the physical distance you cross on the board

Seeing, thinking, speaking, listening, moving the pawn - the student experiences all these things while playing the game. This is how *number sense* develops. Of course there are other programmes that use movement and multi-sensory experience for teaching purposes. But what makes the Great Race different is that it is restricted to the numbers 1 to 10. And because it is played individually it can also be played at home.

4

Chapter 4

KNOWING THE VALUE OF EACH NUMBER

Which number is larger: 5 or 6? The question isn't as simple as it may seem. After all, the digit symbols don't reveal the values involved. Nothing in the shape of the 6 betrays the magnitude it represents. It's just an arbitrary form. As is 5.

The sound 'six' doesn't reveal its meaning either. A toddler picking up this word will often recognize it as a number word. One-two-three-four-five-six-seven-eight-nine-ten; yes, that is the group it belongs to. But the fact that 'six' has a precise content, is knowledge that has to be built.

How should we envision this learning process? A child that learns to count practices the art of encoding. Number words and digits refer to specific magnitudes. The learning machine that is our brain must develop a different magnitude code for each of the small numbers. Without this development, digits and number words remain empty of meaning. This is what the machine must learn:

- When I hear two or see 2, it means • •.
- When I hear three or see 3, it means • • •.
- When I hear six or see 6, it means • • • • • •.

In this way the most important numerical symbols are connected to real, that is non-symbolic, magnitudes. For number sense to develop, these connections must become automatic.

You might say that when children are counting, adding and subtracting, they're sizing up the numbers. The size of two, 2, • •, is smaller than the size of three, 3, • • •. This is consistently the case. So, after a time, you *just know* these values. Your memory creates distinctive codes for every number up to ten, and for some other salient numbers too. These codes enable you to calculate with confidence.

While counting, the child will tell the learning machine that is its brain: 'remember now: this is *one*, this is *two*, this is *three*, you knew that, and the next number is called *four*. Look at the difference: four is one further in the row from three. Four blocks is one more than three blocks. And now you have to start encoding that difference for me reliably. It's no good if, when I say 'four', you give me the same value as for 'three'. Because then I won't be able to rely on my own mind! And that will make things very difficult when I'm doing math.'

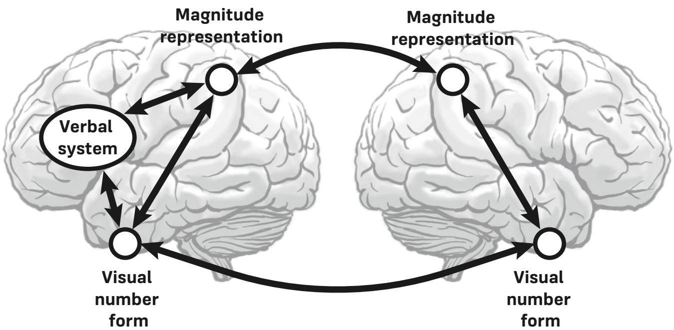
But some brains must work harder and longer than others to establish a distinctive code for each digit.

What is a number? A number is an idea. And you form this idea by combining three things.

- A 'real', perceptible quantity, one you can see or hear or feel, for example • • • • •, or tick-tick-tick-tick-tick
- The matching number word, 'five' in English
- The matching digit, '5' in many languages

Together, these codes make up the number five.

This combination is known as the 'triple code', which the brain must learn to apply.[7] The picture below shows how it works. What you see is a brain viewed from two sides. The hemispheres are depicted back to back. The left hemisphere is pictured on the left, the right hemisphere on the right.



Looking at these two images, you'll appreciate how hard our brains must work to forge that triple code into a single idea for each number. As you see, these three codes are generated in areas of the brain that are pretty far apart.

- The visual number form is the digit '5'. This image is sent from the eyes to the back of the brain. There this '5' is recognized as a form, but not yet as an idea.
- The area named verbal system is the location where the sound 'five' is analysed after it has entered the brain by way of the ears. There, 'five' is recognized as an auditory form: a word.
- And then, very important, there is the 'magnitude representation' at the top. The parietal lobe is specialized in the integration of input from the different senses. In the 'numerical' part of the parietal lobe, the HIPS, the brain integrates number words and digits with its knowledge of 'real' non-symbolic quantities such as • • • • •.

So the brain must learn to make the connections between these three codes effortlessly. When this has been accomplished, each number has acquired its unique, fixed meaning.

So what happens if it isn't accomplished? In that case dyscalculia may develop. A famous case of dyscalculia is presented by neuropsychologist Brian Butterworth in his *book The Mathematical Brain*.[8] There, he tells the story of Charles, a student of psychology at the University of London. Charles could only understand the meaning of numerical symbols and number words by mentally reciting the counting sequence. When asked which number is larger, three or seven, he would count from seven up to ten. If he did not encounter 'three', he concluded that three must be smaller. Fortunately, not all dyscalculic individuals are as seriously handicapped as this. But there is always a lack of number sense.

So how can we help the brain to make these connections?
- Practice regularly with numbers up till ten. Throw a die and ask the student to name the number. The trick is to get the student to practice recognizing and naming them quickly.
- These numbers must be known by their *absolute* magnitude (• • • • •) and by their *relative* magnitude. Ask the student which is the larger number, 3 or 5? And which is smaller: 3 or 2? They need to answer quickly, without counting.
- Use playing cards to practice automatic sequencing. Hold up a card and ask: which number comes before this one? And which number comes after? The answers need to keep coming faster and faster.

Don't make it too difficult. The temptation will quickly arise when things are going well. But don't fall for it. Give the student room to enjoy the achievement.